

Extensible datatypes

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Overview

1 Motivation

- Lightweight generic programming
- Other applications of extensible datatypes

2 Open datatypes and functions

- Open datatypes
- Open functions
- Open problems

3 Related work

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Generic programming via representation types

- Hinze and Cheney: *A lightweight implementation of generics and dynamics*
- Also: Hinze's *Fun of Programming* chapter
- Idea: a value of type $\text{Rep } a$ is a representation of type a ; then use value-level pattern matching to define functions that require a type-level case construct

Implementation of representation types

- 1 pairs of isomorphisms

```
data Iso :: * → * → * where I :: (a → b) → (b → a) → Iso a b
```

- 2 equality types

```
data Eq :: * → * → * where P :: (∀ f. f a → f b) → Eq a b
```

- 3 GADTs

```
data Eq :: * → * → * where P :: Eq a a
```

Example

data Rep :: * → * **where**

$R_{Unit} :: \text{Rep } ()$

$R_{Int} :: \text{Rep } \text{Int}$

$R_{Char} :: \text{Rep } \text{Char}$

$R_{\text{Either}} :: \text{Rep } a \rightarrow \text{Rep } b \rightarrow \text{Rep } (\text{Either } a \ b)$

$R_{(,)} :: \text{Rep } a \rightarrow \text{Rep } b \rightarrow \text{Rep } (a, b)$

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 $R_{(,)} :: \text{Rep } a \rightarrow \text{Rep } b \rightarrow \text{Rep } (a, b)$

$eq :: \text{Rep } a \rightarrow a \rightarrow a \rightarrow \text{Bool}$
 $eq \ R_{Unit} \quad () \quad () \quad = \ \text{True}$
 $eq \ R_{Int} \quad n_1 \quad n_2 \quad = \ n_1 \equiv n_2$
 $eq \ R_{Char} \quad c_1 \quad c_2 \quad = \ c_1 \equiv c_2$
 $eq \ (R_{\text{Either}} \ r_a \ r_b) \ (Left \ a_1) \ (Left \ a_2) \ = \ eq \ r_a \ a_1 \ a_2$
 $eq \ (R_{\text{Either}} \ r_a \ r_b) \ (Right \ b_1) \ (Right \ b_2) \ = \ eq \ r_b \ b_1 \ b_2$
 $eq \ (R_{\text{Either}} \ r_a \ r_b) \ _ \quad _ \quad = \ \text{False}$
 $eq \ (R_{(,)} \ r_a \ r_b) \ (a_1, b_1) \ (a_2, b_2) \ = \ eq \ r_a \ a_1 \ a_2 \ \wedge \ eq \ r_b \ b_1 \ b_2$

Evaluation of the approach

This approach seems to have a number of advantages over other generic programming approaches, such as Generic Haskell:

- Lightweight, only modest extensions required. Implemented in GHC.
- Value-level constructs can be reused (pattern matching, recursion).
- Generic functions are first-class.

Higher-order generic functions

type $GT = \forall a. \text{Rep } a \rightarrow a \rightarrow a$

$bu :: GT \rightarrow \text{Rep } a \rightarrow a \rightarrow a$

$bu \ g \ R_{Unit} \ () = g \ R_{Unit} \ ()$

...

$bu \ g \ (R_{(,)} \ r_a \ r_b) \ (a, b) = g \ (R_{(,)} \ r_a \ r_b) \ (bu \ g \ r_a \ a, bu \ g \ r_b \ b)$

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$incAge :: GT$

$incAge \ R_{Int} \ n = n + 1$

$incAge \ _ \ x = x$

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$bu \ incAge \ db$

Extensibility?

What if

- we want to apply a generic function to a new type that isn't expressible in terms of `Rep` (such as `(→)` or `IO` or any other abstract type)?
- the behaviour of a generic function on a specific datatype should not follow the generic pattern?

Extensibility?

What if

- we want to apply a generic function to a new type that isn't expressible in terms of `Rep` (such as `(→)` or `IO` or any other abstract type)?
- the behaviour of a generic function on a specific datatype should not follow the generic pattern?

Two possibilities:

- 1 Define a new representation datatype.
- 2 Extend the `Rep` datatype.

Define a new representation datatype

- Probably shares a lot of code with the original Rep type.
- We need to convert between real datatypes and their representations for all representation types.
- Most generic functions will use different representation types.
- Higher-order generic functions are not feasible, because they're tied to one particular representation type.

Extend the Rep datatype

- This is the solution usually taken in the papers.
- It is usually required to adapt the functions such as *eq* and *bu*, too.

Example

Extend the `Rep` type with a case *Embed* to represent datatypes that can be encoded using the other constructors.

```
data Rep :: * → * where
```

```
...
```

```
Embed :: Iso a b → Rep b → Rep a
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An example of such a type is *Bool*:

```
rBool = Embed isoBool (REither RUnit RUnit)
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An example of such a type is *Bool*:

```
rBool = Embed isoBool (R_Either R_Unit R_Unit)
```

The `eq` function can be extended to work with embedded types.

```
eq :: Rep a → a → a → Bool  
eq ...  
eq (Embed (I i_a→b i_a→b) r_b) a1 a2 = eq r_b (i_a→b a1) (i_a→b a2)
```

Example – continued

Define a new constructor for a specific datatype:

```
data Rep :: * → * where
```

```
...
```

```
RBool :: Rep Bool
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Example – continued

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Now we can give a specific behaviour of *eq* for *Bool*:

```
eq Bool a1 a2 = False
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Example – continued

Define a new constructor for a specific datatype:

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data Rep :: * → * where
```

```
...
```

```
RBool :: Rep Bool
```

Now we can give a specific behaviour of *eq* for *Bool*:

```
eq Bool a1 a2 = False
```

We can also assign a default behaviour to *eq*:

```
embed :: Rep a → Rep a
```

```
embed RBool = rBool
```

```
eq :: Rep a → a → a → Bool
```

```
eq ...
```

```
eq ra a1 a2 = eq (embed ra) a1 a2
```

Extend the Rep datatype – continued

- Not supported by Haskell, because datatypes are closed.
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Extend the Rep datatype – continued

- Not supported by Haskell, because datatypes are closed.
- We have to rewrite code that is scattered across multiple places and modules.

As a result, it is not possible to

- define a library for generic programming in this style
- use this encoding as back-end for a language such as Generic Haskell, where we want to support separate compilation

Type classes?

Type classes are open.

However,

- defining one class per generic function leads to generic functions that are not first-class citizens anymore/again
- defining generic functions via Hinze's *Generics for the masses* does not solve the extensibility problem

Generics for the masses

Definition of equality:

```
newtype Equality a = Equality { applyEquality :: a → a → Bool }
```

```
instance Generic Equality where
```

```
  unit  = Poly (λ() () → True)
```

```
  int   = Poly (λn1 n2 → n1 ≡ n2)
```

```
  char  = ...
```

```
  either = ...
```

```
  pair  = ...
```

```
eq :: (Rep a) ⇒ a → a → Bool
```

```
eq      = applyEquality rep
```

Generics for the masses

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```

```
eq :: (Rep a) ⇒ a → a → Bool
```

```
eq      = applyEquality rep
```

- Pro: One representation class.
- Contra: The “cases” of the generic function are the methods of the class. Classes cannot be extended with new methods.

Other applications: Compilers

open Expr :: * **where**

Const :: Int → Expr

Add :: Expr → Expr → Expr

eval :: Expr → Int

eval (*Const* *n*) = *n*

eval (*Add* *x* *y*) = *eval* *x* + *eval* *y*

Other applications: Compilers

open Expr :: * **where**

Const :: Int → Expr

Add :: Expr → Expr → Expr

eval :: Expr → Int

eval (*Const* *n*) = *n*

eval (*Add* *x* *y*) = *eval* *x* + *eval* *y*

open Expr :: * **where**

Neg :: Expr → Expr

eval (*Neg* *x*) = *negate* (*eval* *x*)

Again, we have to extend both the datatype and the function.

Other applications: Compilers – continued

This style of programming with open types is similar to AG programming:

```
data Expr | Const (n : Int)
          | Add (x : Expr) (y : Expr)
attr Expr | eval : syn Int
sem Expr | Const lhs.eval = n
          | Add lhs.eval = x.eval + y.eval
```

Other applications: Compilers – continued

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```
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attr Expr | eval : syn Int
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          | Add lhs.eval = x.eval + y.eval
```

```
data Expr | Neg (x : Expr)
sem Expr | Neg lhs.eval = x.eval
```

There is, however, no direct correspondence for inherited attributes.

Other applications: Exceptions

open Exception

throwIO :: Exception → IO a

catch :: IO a → (Exception → IO a) → IO a

Whenever a new form of exception is needed, we can add a new constructor to the Exception type.

Other applications: Exceptions

open Exception

$throwIO :: \text{Exception} \rightarrow IO\ a$

$catch :: IO\ a \rightarrow (\text{Exception} \rightarrow IO\ a) \rightarrow IO\ a$

Whenever a new form of exception is needed, we can add a new constructor to the Exception type.

The *catch* construct is generally used as follows:

```
catch (expr) ( $\lambda e \rightarrow$  case e of  
    SomeException ...  $\rightarrow$  ...  
    –  $\rightarrow$  throw e)
```


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Goals and problems

Goals:

- We want open datatypes and functions.
- We want extensibility across multiple modules.

Goals and problems

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- We want open datatypes and functions.
- We want extensibility across multiple modules.

Problems:

- How to deal with export/import restrictions?
- How to deal with pattern matching?
- What about type inference?
- How to implement open functions?

Open datatypes

An *open datatype* is defined as follows:

```
open TypeName :: kind where  
  Constructor1 :: ...  
  ...
```

- Multiple declarations for the same `TypeName` are possible.
- The name `TypeName` is in the same namespace as all other datatypes.
- The definition defines a new datatype if `TypeName` is not yet in scope, it extends the datatype if `TypeName` is already in scope.
- Implementation is probably less efficient than for closed datatypes, but not really problematic.

Open functions

Functions on open datatypes are open, too. Consider *eval*:

```
eval :: Expr      → Int  
eval (Const n) = n  
eval (Add x y) = eval x + eval y
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eval (Neg x) = negate (eval x)
```

- Open functions require a type signature.
- If a type signature contains an open type, the function is open.

Implementing open functions – pattern matching

- Haskell's linear pattern matching is a problem for open functions, because it is hard to define a linear order between different places where the function is defined.

```
module M where { f :: ... }  
module I where { import M; f ... = ... }  
module J where { import M; f ... = ... }  
module X where { import I; import J }
```

- Even if we had a well-defined order, specifying a default case would effectively close the function.

```
eval _ = 0
```


Best-fit rather than first-fit

The solution is similar to the approach taken for overlapping class instances.

- All branches of a function definition must have the same number of arguments (already the case in Haskell 98).
- The left-most best match is selected.

Therefore:

- Each partial definition of an open function contributes a list of cases/rules.
- The cases are combined/ordered using the above rules for pattern matching.

Example of best-fit pattern matching

```
f :: [Int] → Either Int Char → ...
```

```
f (x : xs) (Left 1)
```

```
f y (Right a)
```

```
f (0 : xs) (Right 'X')
```

```
f [1] z
```

```
f [0] z
```

```
f [] z
```

```
f [0] (Left b)
```

```
f [0] (Left 2)
```

```
f y z
```

```
f [x] z
```

Example of best-fit pattern matching

$f :: [\text{Int}] \rightarrow \text{Either Int Char} \rightarrow \dots$

$f \ (x : xs) \ (\text{Left } 1)$

$f \ y \ (\text{Right } a)$

$f \ (0 : xs) \ (\text{Right } 'X')$

$f \ [1] \ z$

$f \ [0] \ z$

$f \ [] \ z$

$f \ [0] \ (\text{Left } b)$

$f \ [0] \ (\text{Left } 2)$

$f \ y \ z$

$f \ [x] \ z$

$f :: [\text{Int}] \rightarrow \text{Either Int Char} \rightarrow \dots$

$f \ [] \ z$

$f \ [0] \ (\text{Left } 2)$

$f \ [0] \ (\text{Left } b)$

$f \ [0] \ z$

$f \ (0 : xs) \ (\text{Right } 'X')$

$f \ [1] \ z$

$f \ [x] \ z$

$f \ (x : xs) \ (\text{Left } 1)$

$f \ y \ (\text{Right } a)$

$f \ y \ z$

Implementing open functions – recursion

- An open function is implicitly parametrized over the final closed version of the function.

$eval :: (eval) \Rightarrow Expr \rightarrow Int$

Intermediate code:

$eval \quad eval' \quad (Const \ n) = n$
 $eval \quad eval' \quad (Add \ x \ y) = eval' \ x + eval' \ y$

- Other functions that make use of open functions inherit these implicit parameters.

$f = \dots eval \ something \dots$

\rightsquigarrow

$f \ eval' = \dots eval' \ something \dots$

Remaining problems

- Is best-fit pattern matching sufficient for all cases? (Should be for the given examples.)
- First-class rules might be an alternative for best-fit pattern matching.

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- What about open datatypes and **deriving**/generic functions?
- Can we define a transformation between type classes and extensible datatypes?
- Open datatypes and functions are closed once, for the whole program. Would it be beneficial to allow to close them earlier, or multiple times? (Related to subtyping.)

Related work

- Type classes.
- GADTs.
- First-class patterns and rules.
- Subtyping.
- Extensible records.